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Heat transfer during asymmetrical collision of thermal waves in a thin film

ZHI-MING TAN

Chemical Engineering Research Institute, South China University of Technology, Guangzhou 510641, People's Republic of China

and

WEN-JEI YANG†

Thermal-Fluids Laboratory, Department of Mechanical Engineering and Applied Mechanics, The University of Michigan, Ann Arbor, MI 48109, U.S.A.

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Abstract—Wave nature of heat propagation in a thin film subjected to an asymmetrical temperature change on both sides is investigated by solving the hyperbolic heat conduction equation. Analytical expressions for the temperature and heat flux distributions and numerical results for the time history of heat transfer behavior are obtained. It is disclosed that in transient heat conduction, a heat pulse is transported as a wave, which is attenuated in the film, and that non-Fourier heat conduction is extremely significant within a certain range of film thickness and time. When a film with smaller value of $x_0/(\tau C)$ is heated on both side walls, temperature overshoot occurs within a very short period of time. © 1997 Elsevier Science Ltd.

1. INTRODUCTION

The constitutive relation which appears in classical heat conduction is Fourier's law:

$$q = -K\frac{\partial T}{\partial x}. \tag{1}$$

Here, q denotes the heat flux, T is the temperature, x is the distance from the left side wall of the film and K is the thermal conductivity of the medium. Fourier's law predicts that conduction is a diffusion phenomenon in which temperature disturbances propagate at an infinite velocity and at time $t = 0+$, the heat flux at the wall is infinite, while the temperature change is nonzero everywhere except at infinity. This phenomenon is physically anomalous and can be remedied through the introduction of a hyperbolic equation based on a relaxation model for heat conduction that accounts for a finite thermal propagation speed.

In most practical heat transfer applications, the effect of a finite speed of propagation is negligible since materials in which heat propagates are macroscopic in dimension such that Fourier's law is accurate and appropriate. However, this law noticeably breaks down in situations involving very short times, high heat fluxes and at cryogenic temperatures, because the wave nature of heat propagation becomes dominant [1–5]. Several issues of basic scientific interest arise in cases such as laser penetration and welding, explosive bonding, electrical discharge machining, and heating and cooling of micro-electronic elements involving a duration time of nanosecond or even picosecond in which energy is absorbed within a distance of microns from the surface. For example, the issue of energy transfer into a lattice and resulting temperature in the lattice during such a short period of time and over such a tiny region is of fundamental importance, but remains a matter of controversy [6]. It is apparent that a more accurate constitutive law describing the nature of heat conduction needs to be introduced.

Recently, considerable interest has been generated toward the hyperbolic heat conduction equation and its potential applications in engineering and technology. A comprehensive survey of the pertinent literature is available in [7]. Theoretical predictions are available in the literature for some specific cases. Some dealt with wave characteristics and finite propagation speed in transient heat transfer conduction [4, 5, 8–14]. The hyperbolic heat equation was used by Baumeister and Hamill [15] to study the propagation of a temperature pulse in a semi-infinite medium, and by Vick and Ozisik [13] and Ozisik and Vick [16] to study the propagation of a heat pulse. The present authors [17] investigated heat transfer resulting from symmetrical collision of thermal waves induced by a step change in the wall temperature of a thin film. Results were obtained for the time history of propagation process, magnitude and shape of thermal waves and the range of film thickness and duration time.

This paper treats the wave behavior during transient heat conduction in a very thin film (solid plate)

† Author to whom correspondence should be addressed.

NOMENCLATURE

a	separation constant, $a_i = i\pi/[x_0/(\tau C)]$ and $i = 1, 2, \ldots$
A	a function of dimensionless time β
B	a function of dimensionless distance δ
C	thermal propagation speed
C_p	specific heat
K	thermal conductivity
q	heat flux
q_m	heat flux defined as $q_m = KT_0x_0$
Q	the amount of heat
Q_m	the amount of heat the film can absorb at steady state, $Q_m = 3C_p\rho x_0T_0/2$
T	temperature
T_0	initial temperature
T_1	left side wall temperature of the film
T_2	right side wall temperature of the film
t	time
x	distance
x_0	half thickness of the film.

Greek symbols

α	thermal diffusivity
β	dimensionless time defined as $t/(2\tau)$
δ	dimensionless distance defined as $x/(2\tau C)$
ρ	density
τ	thermal relaxation time
θ	dimensionless temperature defined as $[2x_0(T_1-T_0)-x(T_2-T_1)]/(2x_0T_0)$.

subjected to an asymmetrical temperature change on both side surfaces. Analytical solutions are obtained by means of the method of separation of variables to solve the non-Fourier, hyperbolic-type heat conduction equation.

2. FORMULATION OF THE PROBLEM AND SOLUTIONS

The modified Fourier equation [7] can be expressed as:

$$\tau\frac{\partial q}{\partial t}+q+K\frac{\partial T}{\partial x}=0 \qquad (2)$$

which is the constitutive equation used in linearized thermal wave theory. Here, τ denotes the relaxation time $\tau = \alpha/C^2$ where C is the speed of 'second sound' (thermal shock wave) and α represents the thermal diffusivity of the medium. The thermal propagation speed C becomes finite for $\tau > 0$. As τ approaches zero, the thermal propagation speed C approaches infinity and the classical parabolic law is recovered.

In one-dimensional flow of heat, the conservation of energy is given by:

$$\rho C_p\frac{\partial T}{\partial t}+\frac{\partial q}{\partial x}=0. \qquad (3)$$

Here, ρ and C_p denote the density and specific heat of the medium, respectively. A combination of equation (2) and (3) yields the hyperbolic conduction equation as:

$$\tau\frac{\partial^2 T}{\partial t^2}+\frac{\partial T}{\partial t}=\alpha\frac{\partial^2 T}{\partial x^2} \qquad (4)$$

Consider a very thin film with a thickness of $2x_0$ maintained at a uniform, initial temperature T_0. The walls at $x = 0$ and $x = 2x_0$ are suddenly raised to a temperature T_1 and T_2, respectively. Thus, the initial and boundary conditions are:

$$T = T_0 \quad \text{at } t=0 \quad 0<x<2x_0 \qquad (5)$$

$$\partial T/\partial x = 0 \quad \text{at } t=0 \quad 0<x<2x_0 \qquad (6)$$

$$T = T_1 \quad \text{at } t>0 \quad x=0 \qquad (7)$$

$$T = T_2 \quad \text{at } t>0 \quad x=2x_0. \qquad (8)$$

With the introduction of the following dimensionless quantities:

$$\theta(\beta,\delta)=\frac{(T-T_1)}{T_0}-\frac{(T_2-T_1)x}{2x_0T_0}$$

$$\beta=\frac{t}{2\tau}$$

$$\delta=\frac{x}{2\tau C}.$$

Equations (4)–(8) are reduced to:

$$\frac{\partial^2\theta}{\partial\beta^2}+2\frac{\partial\theta}{\partial\beta}=\frac{\partial^2\theta}{\partial\delta^2} \qquad (9)$$

$$\theta=[2x_0(T_0-T_1)-x(T_2-T_1)]/(2x_0T_0)$$
$$\text{at } \beta=0 \quad 0<\delta<x_0/(\tau C) \qquad (10)$$

$$\partial\theta/\partial\beta=0 \quad \text{at } \beta=0 \quad 0<\delta<x_0/(\tau C) \qquad (11)$$

$$\theta=0 \quad \text{at } \beta>0 \quad \delta=0 \qquad (12)$$

$$\theta=0 \quad \text{at } \beta>0 \quad \delta=x_0/(\tau C). \qquad (13)$$

An application of the method of separation of variables using:

$$\theta(\beta,\delta)=A(\beta)B(\delta). \qquad (14)$$

One obtains the general solutions for the heat equation (9) as:

$$\theta = [D\cos(\sqrt{a^2-1}\cdot\beta)+E\sin(\sqrt{a^2-1}\cdot\beta)]e^{-\beta} \times [H\cos(a\cdot\delta)+J\sin(a\cdot\delta)] \quad \text{at } a>1. \quad (15)$$

$$\theta = (Fe^{\sqrt{1-a^2}\cdot\beta}+Ge^{\sqrt{1-a^2}\cdot\beta})e^{-\beta} \times [H\cos(a\cdot\delta)+J\sin(a\cdot\delta)] \quad \text{at } a<1. \quad (16)$$

Here, a is a constant and D, E, F, G, H and J are the coefficients to be determined by the initial and boundary conditions. In the interest of brevity, the intermediate steps of derivation are omitted here. The general solutions, thus obtained, are recast in the dimensional expression for the temperature time history as:

(a) for $x_0/(\tau C) < \pi$

$$T = \frac{(T_2-T_1)\cdot\delta}{x_0/(\tau C)} + T_1 + \sum_{i=1}^{\infty}\frac{2\cdot[(T_0-T_1)+(T_2-T_0)\cdot(-1)^i]}{i\cdot\pi} \cdot\left(\cos(\sqrt{a_i^2-1}\cdot\beta)+\frac{1}{\sqrt{a_i^2-1}} \cdot\sin(\sqrt{a_i^2-1}\cdot\beta)\right)\cdot e^{-\beta}\cdot\sin(a_i\cdot\delta) \quad (17)$$

(b) for $x_0/(\tau C) > (i-1)\pi$

$$T = \frac{(T_2-T_1)\cdot\delta}{x_0/(\tau C)} + T_1 + \sum_{i=1}^{\text{int}\left(\frac{x_0}{\pi\tau C}-\frac{1}{2}\right)}\frac{(T_0-T_1)+(T_2-T_0)\cdot(-1)^i}{i\cdot\pi} \cdot\left(\frac{\sqrt{1-a_i^2}+1}{\sqrt{1-a_i^2}}\cdot e^{\sqrt{1-a_i^2}\cdot\beta}+\frac{\sqrt{1-a_i^2}-1}{\sqrt{1-a_i^2}}\cdot e^{\sqrt{1-a_i^2}\cdot\beta}\right) \cdot e^{-\beta}\cdot\sin(a_i\cdot\delta) + \sum_{i=\text{int}\left(\frac{x_0}{\pi\tau C}+\frac{1}{2}\right)}^{\infty}\frac{2\cdot[(T_0-T_1)+(T_2-T_0)\cdot(-1)^i]}{i\cdot\pi} \cdot\left(\cos(\sqrt{a_i^2-1}\cdot\beta)+\frac{1}{\sqrt{a_i^2-1}}\cdot\sin(\sqrt{a_i^2-1}\cdot\beta)\right) \cdot e^{-\beta}\cdot\sin(a_i\cdot\delta). \quad (18)$$

Here, $a_i = i\pi/[x_0/(\tau C)]$ and i is an integer $(1, 2, \ldots)$.

Equation (2) is solved for the time history of heat flux as:

$$q(x,t) = \frac{K}{\tau}\cdot e^{-t/\tau}\cdot\int_0^t\left(e^{t/\tau}\cdot\frac{\partial T}{\partial x}(x,t)\right)dt. \quad (19)$$

A substitution of equations (17) and (18) into equation (19) yields:

(a) for $x_0/(\tau C) < \pi$

$$\frac{q}{q_m} = \frac{T_2-T_1}{2T_0}\cdot(e^{-2\beta}-1) - \sum_{i=1}^{\infty}\frac{2\cdot[(T_0-T_1)+(T_2-T_0)\cdot(-1)^i]}{T_0\cdot\sqrt{a_i^2-1}} \cdot\sin(\sqrt{a_i^2-1}\cdot\beta)\cdot e^{-\beta}\cdot\cos(a_i\cdot\delta) \quad (20)$$

(b) for $x_0/(\tau C) > (i=1)\pi$

$$\frac{q}{q_m} = \frac{T_2-T_1}{2T_0}\cdot(e^{-2\beta}-1) - \sum_{i=1}^{\text{int}\left(\frac{x_0}{\pi\tau C}-\frac{1}{2}\right)}\frac{(T_0-T_1)+(T_2-T_0)\cdot(-1)^i}{T_0\cdot\sqrt{1-a_i^2}} \cdot(e^{\sqrt{1-a_i^2}\cdot\beta}-e^{\sqrt{1-a_i^2}\cdot\beta})\cdot e^{-\beta}\cdot\cos(a_i\cdot\delta) - \sum_{i=\text{int}\left(\frac{x_0}{\pi\tau C}+\frac{1}{2}\right)}^{\infty}\frac{2\cdot[(T_0-T_1)+(T_2-T_0)\cdot(-1)^i]}{T_0\cdot\sqrt{a_i^2-1}} \cdot\sin(\sqrt{a_i^2-1}\cdot\beta)\cdot e^{-\beta}\cdot\cos(a_i\cdot\delta). \quad (21)$$

Here, $q_m = KT_0/x_0$.

The amount of heat flowing through a cross-section perpendicular to the axis x can be determined using the expression:

$$Q(x,t) = \int_0^t q(x,t)\cdot dt. \quad (22)$$

Substituting equations (20) and (21) into equation (22), one gets:

(a) for $x_0/(\tau C) < \pi$

$$\frac{Q}{Q_m} = \frac{T_2-T_1}{3T_0\cdot(x_0/\tau C)^2}\cdot(1-2\beta-e^{-2\beta}) - \sum_{i=1}^{\infty}\frac{8\cdot[(T_0-T_1)+(T_2-T_0)\cdot(-1)^i]}{3T_0\cdot i^2\cdot\pi^2} \cdot\left(1-e^{-\beta}\cdot\cos(\sqrt{a_i^2-1}\cdot\beta)-\frac{1}{\sqrt{a_i^2-1}} \cdot e^{-\beta}\cdot\sin(\sqrt{a_i^2-1}\cdot\beta)\right)\cdot\cos(a_i\cdot\delta) \quad (23)$$

(b) for $x_0/(\tau C) > (i-1)\pi$

$$\frac{Q}{Q_m} = \frac{T_2-T_1}{3T_0\cdot(x_0/\tau C)^2}\cdot(1-2\beta-e^{-2\beta}) - \sum_{i=1}^{\text{int}\left(\frac{x_0}{\pi\tau C}-\frac{1}{2}\right)}\frac{4\cdot[(T_0-T_1)+(T_2-T_0)\cdot(-1)^i]}{3T_0\cdot i^2\cdot\pi^2} \cdot\left(2-\frac{\sqrt{1-a_i^2}+1}{\sqrt{1-a_i^2}}\cdot e^{\sqrt{1-a_i^2}\cdot\beta}-\frac{\sqrt{1-a_i^2}-1}{\sqrt{1-a_i^2}}\cdot e^{-\sqrt{1-a_i^2}\cdot\beta}\right)$$

$$\cdot\cos(a_i\cdot\delta)-\sum_{i=\mathrm{int}\left(\frac{x_0}{\pi\tau C}+\frac{1}{2}\right)}^{\infty}\frac{8\cdot[(T_0-T_1)+(T_2-T_0)\cdot(-1)^i]}{3T_0\cdot i^2\cdot\pi^2}$$

$$\cdot\left(1-\mathrm{e}^{-\beta}\cdot\cos(\sqrt{a_i^2-1}\cdot\beta)-\frac{1}{\sqrt{a_i^2-1}}\right)$$

$$\cdot\mathrm{e}^{-\beta}\cdot\sin(\sqrt{a_i^2-1}\cdot\beta)\cdot\cos(a_i\cdot\delta). \quad (24)$$

Here, $Q_m = 3\rho C_p x_0 T_0/2$.

3. RESULTS AND DISCUSSION

Numerical results are obtained which display the unusual nature of hyperbolic heat conduction in the films with different values of $x_0/(\tau C)$. For convenience in analysis and computation, a set of $T_2 = 2T_0$ and $T_1 = 1.5T_0$ is selected to demonstrate heat transfer characteristics resulting from heating a film at both side surfaces.

Figures 1–5 show the temperature–time history (T/T_0) in the films having $x_0(\tau C)$ of 1, 2, 3 5 and 10, respectively. One can see that the wave nature in heat conduction involves more than a simple switch from a parabolic equation to a hyperbolic equation. Figure 1 is prepared to illustrate, in detail, the propagation process of thermal waves in a film with the value of $x_0/\tau C$ being 1. Clearly, the same as other wave phenomena, sharp wavefronts exist in the thermal wave propagation while the temperature levels decrease when the thermal waves penetrate into the medium. At $\beta = 0$, the temperature in the film is equal to its initial temperature T_0 [namely $(T-T_0)/T_0 = 0$] and the temperature on two sides are suddenly raised to $1.5T_0$ and $2T_0$, respectively. The temperature distribution in the film is expressed by curve A1′1B at this moment. Then, two wavefronts appear and advance towards the center in the physical domain which separates the heat affected zone from the thermally undisturbed zone. Across the wavefronts, the temperature presents a finite jump. At $\beta = 0.5$, thermal wavefronts meet and collide with each other at the center of the film with a temperature distribution curve A4B in Fig. 1(a). After the first collision, the center temperature undergoes a significant increase resulting a much higher temperature in this region [see curve A5′5B in Fig. 1(b)] while reverse thermal wavefronts occur and travel towards side walls of the film. One note that the field temperature is allowed to exceed the imposed wall temperature while the heat flux has a step decrease when thermal wavefronts reach at the side walls at $\beta = 1.0$ [see curve A8′8B in Fig. 1(b)]. After thermal wavefronts are reflected from the boundaries, the pattern is continued in Fig. 1(c)–(e). By several times of collision, reflection and continuous attenuation of the thermal waves as they propagate back and forth between the two boundaries, the wavefronts become weak and the results predicted by the wave theory collapse onto those predicted by the diffusion model at $\beta = 5.0$ and thereafter.

Similarly, one can see in other figures that an asymmetrical temperature change on two sides of a film gives rise to the propagation of two severe thermal wavefronts in the film at a finite velocity. Each of these wavefronts decays exponentially with time and simultaneously dissipates heat along its path by diffusion. One notices an undisturbed region $t/(2\tau) < \delta < (2x_0 - Ct)/(2\tau C)$ of no temperature change when the dimensionless time β is less than $x_0/(2\tau C)$. The local temperatures at locations $\delta = t(2\tau)$ and $\delta = (2x_0 - Ct)/(2\tau C)$ exhibit a step discontinuity similar to that of semi-infinite body problem [15, 18].

The present analytical solution predicts the existence of thermal waves in a very thin film and exhibits the propagation process of thermal waves, the magnitude and shape of thermal waves, and the regularity of thermal wave decaying process in the films with different values of $x_0/(\tau C)$. Such behavior is characteristic of a thermal system with a relaxation or start up time unseen in the classical linear or nonlinear diffusion theory. It is seen in each figure that the thermal wavefronts from two sides of the film collide at the centre of the film, with the peak of each wave decaying exponentially with time up to $\beta = 5$ (namely $t = 10\tau$). Of particular interest is to observe a temperature overshoot in the films at a smaller value of $x_0(\tau C)$ over a very short period of time, which is induced by the collision of wavefronts. it is found that the non-Fourier effect is far more significant in a system with a larger relaxation time τ. For example, larger temperature waves can be seen in a film with $x_0/(\tau C)$ of unity. It is characterized by a film temperature rising rapidly and greatly exceeding the side wall temperature, called temperature overshoot. The temperature increases rapidly at first followed by a steady decay. However, in a film with $x_0/(\tau C) = 10$, wavefronts are too weak as they approach the symmetrical center to produce temperature wave and temperature overshoot. It behaves like diffusion domination. For $\tau = 0$, equation (18) is reduced to

$$T=\frac{(T_2-T_1)\cdot x}{2x_0}+T_1$$

$$+\sum_{i=1}^{\infty}\frac{2[(T_0-T_1)+(T_2-T_0)\cdot(-1)^i]}{i\cdot\pi}$$

$$\cdot\mathrm{e}^{-\left(\frac{i\pi}{2}\right)^2\cdot\frac{\alpha t}{x_0^2}}\cdot\sin\left(\frac{i\pi}{2}\cdot\frac{x}{x_0}\right) \quad (25)$$

which is the solution of diffusion mechanism.

Figures 1–5 reveal that the thermal relaxation time τ plays a primary role in deciding a domain to be wave dominating or diffusion dominating. Several investigators estimated the magnitude of thermal relaxation time τ to range from 10^{-10} s for gases at standard conditions to 10^{-14} s for metals [19] with that for

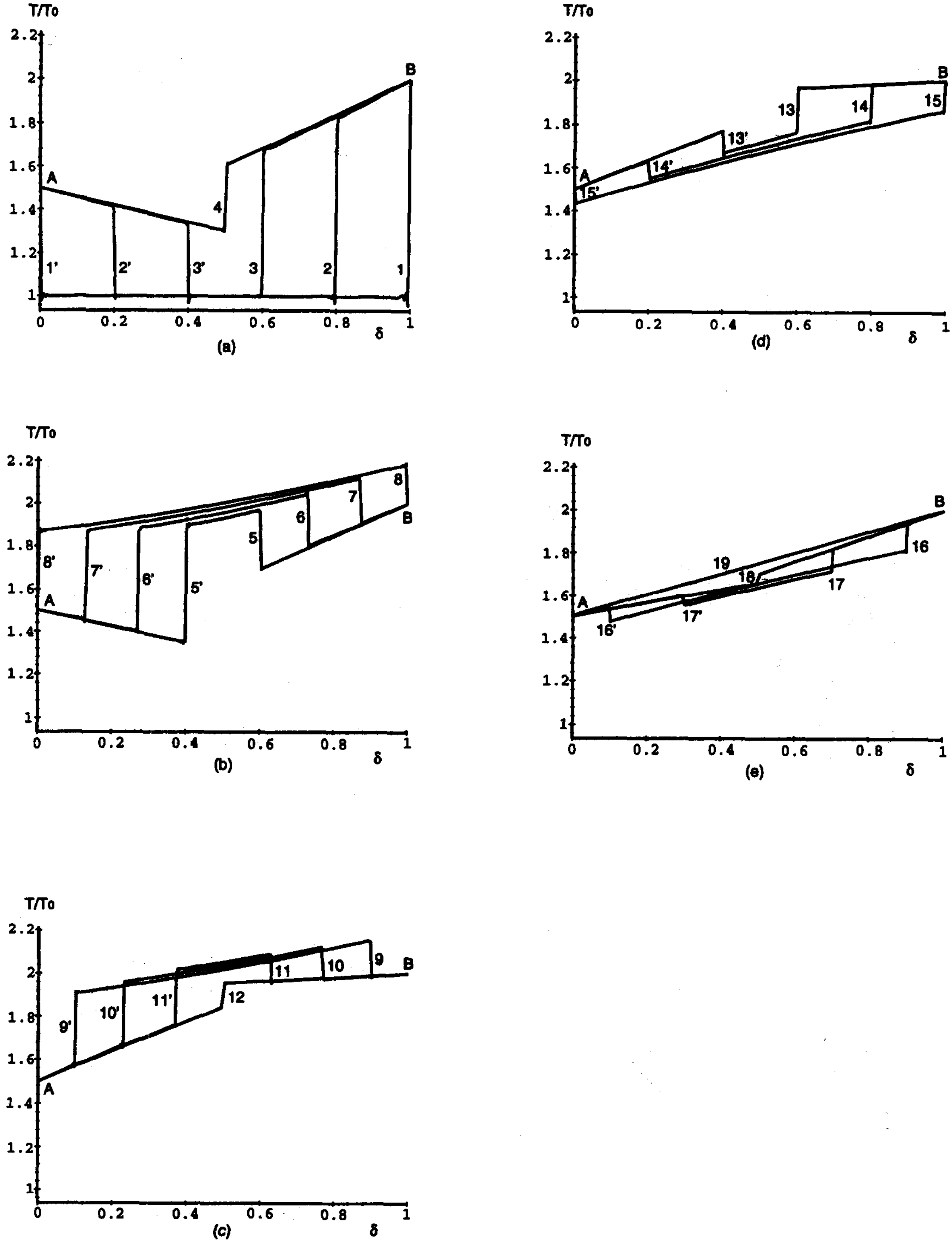

Fig. 1. Instantaneous temperature distributions in the film at $x_0/\tau C = 1$: (1) $\beta = 0$; (2) $\beta = 0.2$; (3) $\beta = 0.4$; (4) $\beta = 0.5$; (5) $\beta = 0.6$; (6) $\beta = 0.73$; (7) $\beta = 0.87$; (8) $\beta = 1.5$; (9) $\beta = 1.1$; (10) $\beta = 1.23$; (11) $\beta = 1.37$; (12) $\beta = 1.5$; (13) $\beta = 1.6$; (14) $\beta = 1.8$; (15) $\beta = 2.0$; (16) $\beta = 2.1$; (17) $\beta = 2.3$; (18) $\beta = 2.5$; (19) $\beta = 5.0$.

liquids [20] and insulators [3] falling within this range. With τ known, one can estimate the range of film thickness within which heat propagates as a wave. The criterion for thermal wave dominating in the present study is $x_0/(\tau C) < 10$, which gives the thickness of the film in the order of about 0.02 micron for silicon.

Figure 6 shows the time history of heat flux q/q_m at the side walls and centre of the films with $x_0/(\tau C) = 1$,

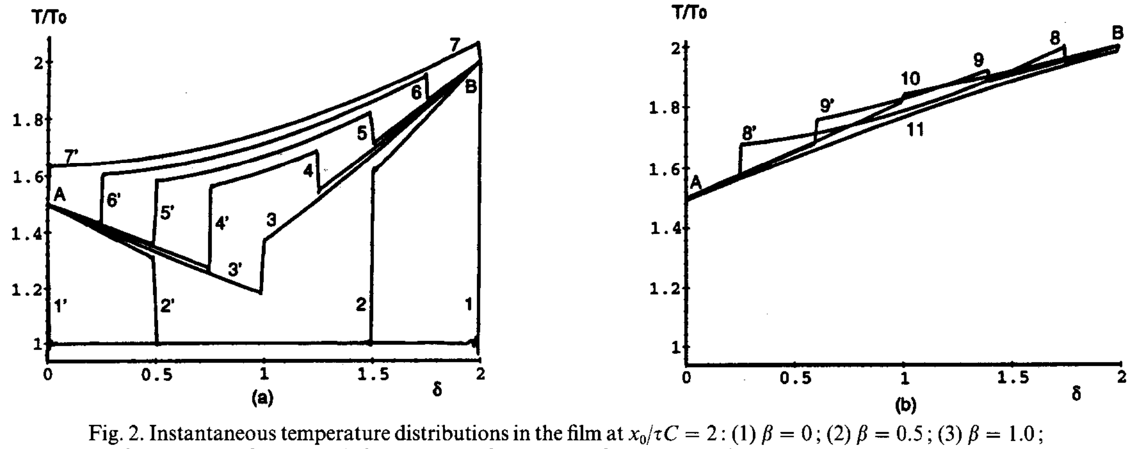

Fig. 2. Instantaneous temperature distributions in the film at $x_0/\tau C = 2$: (1) $\beta = 0$; (2) $\beta = 0.5$; (3) $\beta = 1.0$; (4) $\beta = 1.25$; (5) $\beta = 1.5$; (6) $\beta = 1.75$; (7) $\beta = 2.0$; (8) $\beta = 2.25$; (9) $\beta = 2.6$; (10) $\beta = 3.0$; (11) $\beta = 4.0$.

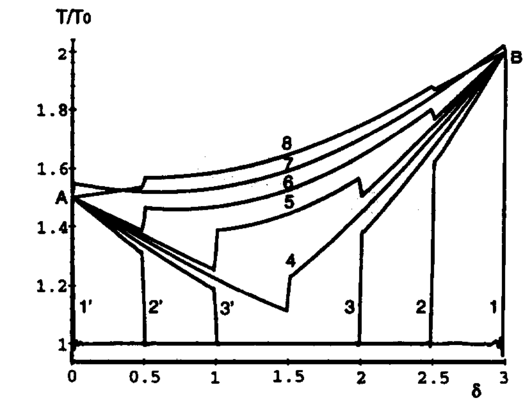

Fig. 3. Instantaneous temperature distributions in the film at $x_0/\tau C = 3$: (1) $\beta = 0$; (2) $\beta = 0.5$; (3) $\beta = 1.0$; (4) $\beta = 1.5$; (5) $\beta = 2.0$; (6) $\beta = 2.5$; (7) $\beta = 3.0$; (8) $\beta = 3.5$.

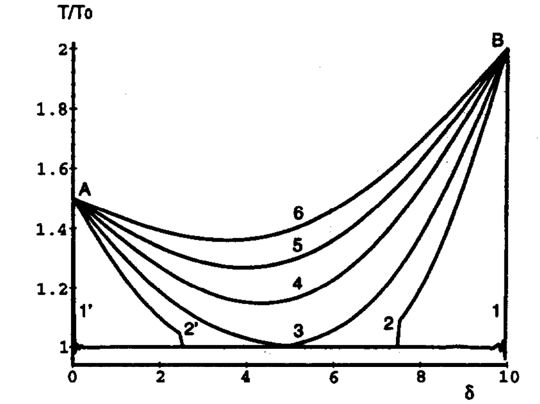

Fig. 5. Instantaneous temperature distributions in the film at $x_0/\tau C = 10$: (1) $\beta = 0$; (2) $\beta = 2.5$; (3) $\beta = 5.0$; (4) $\beta = 10$; (5) $\beta = 15$; (6) $\beta = 20$.

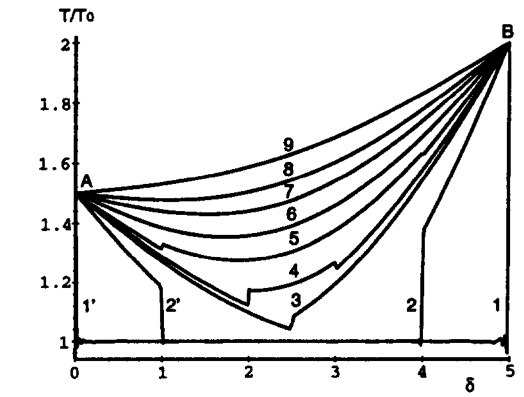

Fig. 4. Instantaneous temperature distributions in the film at $x_0/\tau C = 5$: (1) $\beta = 0$; (2) $\beta = 1.0$; (3) $\beta = 2.5$; (4) $\beta = 3.0$; (5) $\beta = 4.0$; (6) $\beta = 5.0$; (7) $\beta = 6.25$; (8) $\beta = 7.5$; (9) $\beta = 10$.

2 and 3, respectively. It is seen that the absolute values of heat flux at the side walls increase instantly from zero to a maximum upon an introduction of the transient followed by a decrease with time. The maximum value of q/q_m at $\beta = 0+$ is inversely proportional to the relaxation time, giving 0.48, 0.96 and 1.44 at the left side wall of the films with $x_0/(\tau C) = 1$, 2 and 3, respectively. As the relaxation time τ approaches zero, the wall heat flux approaches infinite, in accordance with the parabolic heat conduction theory. Heat flux at the film centre does not occur until thermal wavefronts from both sides of the film meet at the centre of the film. A thermal shock wave is induced at the instant the temperature wave arrives at the side wall. It is followed by a rebound of the thermal wave accompanied by heat loss across the left side wall into the surroundings. When the value of β exceeds 5, no obvious thermal shock waves can be observed with the film almost reaching a steady thermal equilibrium state and uniform heat flux throughout the entire film. In Fig. 6(a), heat loss would occur across the right side wall into the surroundings if the right side wall temperature T_2 could be lower than $2T_0$ or the value of $x_0/(\tau C)$ less than unity.

It is of interest to note in Fig. 6(a) that the first thermal shock wave in curve 1 occurs at an instant of $\beta = t/(2\tau) = 1$, corresponding to:

$$\frac{x_0}{\tau C} = \frac{t}{2\tau} \quad \text{or} \quad C = \frac{2x_0}{t}. \tag{26}$$

Since $2x_0$ represents the film thickness, one can use

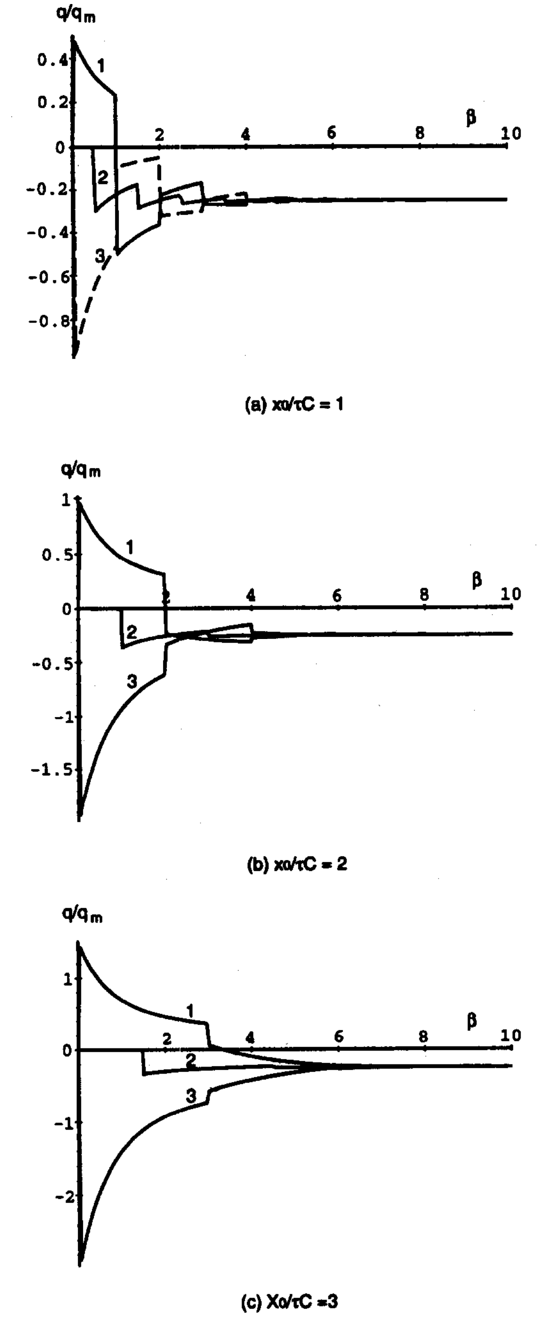

Fig. 6. Time history of heat flux in the films: (a) $x_0/\tau C = 1$; (b) $x_0/\tau C = 2$; (c) $x_0/\tau C = 3$. (1) at left side wall of the film ($\delta = 0$); (2) at the centre of the film ($\delta = x_0/2\tau C$); (3) at right side wall of the film ($\delta = x_0/\tau C$).

the equation to determine the speed of 'second sound' C in the film, by measuring the time interval t followed by evaluating the relaxation time of medium using the expression:

$$\tau = \frac{\alpha}{C^2} = \frac{\alpha \cdot t^2}{4x_0^2}.$$

Figure 7 illustrates the time history of the amount of heat transferred Q/Q_m at the side walls and centre of the films. Figure 7(a) shows that for a film with $x_0/(\tau C) = 1$, the amount of heat flowing into the film from the side walls Q greatly exceeds that the film can absorb at steady-state Q_m within a very short period of time. As a result, the difference in the values of Q/Q_m between curves 1 and 3 exceeds unity at smaller values of β. This difference is 1.35 at $\beta = 1$ when thermal waves reach the side wall the first time, but is reduced to 0.88 at $\beta = 2$ when thermal waves reach the side wall the second time. A thermal equilibrium state is almost established when β exceeds 5, with a constant heat flux. This phenomenon becomes obscure in the film with $x_0/(\tau C) = 3$ in Fig. 7(b).

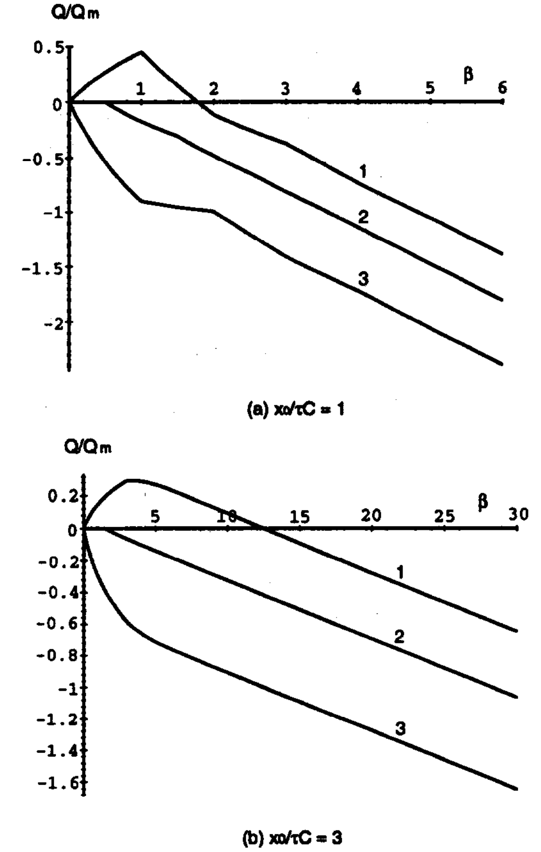

Fig. 7. Time history of total amount of heat transferred in the films: (a) $x_0/\tau C = 1$; (b) $x_0/\tau C = 3$. (1) at left side wall of the film ($\delta = 0$); (2) at the centre of the film ($\delta = x_0/2\tau C$); (3) at right side wall of the film ($\delta = x_0/\tau C$).

4. CONCLUSIONS

Heat wave and hyperbolic heat transfer phenomena have been theoretically studied in a very thin film subjected to an asymmetrical temperature change at the side walls, using a method of separation of variables. Results have been obtained for the propagation process, magnitude and shape of thermal waves and the range of film thickness and duration time of thermal wave propagation.

It is revealed that thermal waves appear only when τC is of the same order as or larger than one half the film thickness, namely $x_0/(\tau C) < 10$. The smaller the value of $x_0/(\tau C)$, the more pronounced the temperature waves. The criterion for the occurrence of thermal shock waves in a thin film is for the film thickness to be in the order of about 0.02 micron for silicon and within the time duration of 10 times the relaxation time. Temperature overshoot may occur in the films of smaller $x_0/(\tau C)$ values within a very short period of time.

Wall heat flux undergoes a sharp change at an introduction of transient and a step change at the moment the rebound temperature waves reach the walls. After a very short period of time ($\beta > 5$), temperature waves disappear and a uniform heat flux is established throughout the film. The shorter the thermal relaxation time, the larger the maximum heat flux at zero time. As the thermal relaxation time approaches zero, the maximum heat flux approaches infinite in accordance with the parabolic heat conduction theory. The study provides a method for the determination of both thermal propagation speed and relaxation time.

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